Improving the Performance of Fading Channel Simulators Using New Parameterization Method

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***Abstract*—The modeling and simulation of mobile fading channels can be done efficiently by using the finite sum of weighted sinusoids with equally distributed phases known as the concept of Rice method. In this paper, we evaluate the statistical properties of Weibull-lognormal fading channel model, such as Level Crossing Rate (LCR), Average Duration of Fades (ADF), and Probability Density Function (PDF). Several results are obtained by using different methods to design the deterministic simulation model parameters. New computation method of deterministic simulation model parameters is presented. This procedure is a combination of two methods, Method of Equal Areas and Method of Exact Doppler Spread. It is called a combination of MEA and MEDS. Comparisons between Autocorrelation Functions (ACFs) of both reference and simulation models are introduced for different methods. It is demonstrated by several simulation results that statistical properties of simulation and reference models will be much closer according to the new method than MEDS and MEA. Finally, the results indicate the superiority of the new method over the MEDS and MEA with respect to LCR and ADF of Weibull-lognormal fading channel model.**

***Index Terms*—level crossing rate, average duration of fades, autocorrelation functions**

I. Introduction

The modeling of fading channels has a great importance in the design, test and improvement the performance of cellular radio communication systems. The channel simulator must be efficient, flexible and accurate. In addition, the statistical properties of the channel simulator should be very close to the statistical behavior of the desired reference model [1]. This will depend on the design method of channel simulator. Depending upon the radio propagation environment, various multipath fading models are available in literature [2]. Both of the advances on classical fading models and a brief summary of some new fading models were presented in [3].

Mobile radio channels are classified into two main categories, namely frequency-nonselective and frequency-selective channels. The first type is modeled by using an appropriate stochastical models, such as Rayleigh, Rice and Suzuki processes [4], whereas frequency-selective channels can be modeled by using (n-path) tap delay line model [5], which requires 2n coloured Gaussian processes. Therefore, computer simulation models can be implemented by means of the Rice method [6], which depends on approximation of the coloured Gaussian processes by finite sum of weighted sinusoids with phases uniformly distributed. Finding proper design method for computing parameters of simulation models provides deterministic processes at the output of channel simulator with a statistical properties closed to those of corresponding stochastic processes, especially statistics of second order such as LCR and ADF. Analytical expressions for these quantities have been derived for Rayleigh and Rice [7], [8]. In this research we are interested in modelling frequency-nonselective channels, where Suzuki process [9] is considered to be a more suitable model for non-selective frequency cellular radio channels in many cases. Suzuki process is obtained by multiplying Rayleigh and Lognormal processes with each other. The reader can find detailed information on the extended and modified stochastic versions of Suzuki models in [9]. It is worth mentioning that these stochastic models are not capable of modeling non–uniform scattering. In order to combine inhomogeneous diffuse scattering with shadow fading, the Weibull-lognormal process appears as an appropriate composite model [10]. The Weibull-lognormal model, consists of three stochastic Gaussian processes, should be designed by a proper method. There are many methods to calculate parameters of simulation model (doppler coefficients and discrete doppler frequencies), for example the Method of Equal Areas (MEA) [1], [9], which provides a satisfied approximation of the desired statistics for Jakes Doppler power spectral density even for a small number of sinusoids, but it fails and requires large number of sinusoids for other types of Doppler power spectral densities such as those with Gaussian shapes [11]. Furthermore, it is found that MEA does not result in a periodic ACF due to unequal distances between discrete Doppler frequencies [12]. There is another method called Method of Exact Doppler Spread (MEDS), which compute parameters in such way the Doppler spread is the same for both reference and simulation models [13]. In this paper, new optimal design method of simulation model parameters is presented. This method is named a combination of MEA and MEDS. Therefore, a brief review of reference models for Rayleigh and Weibull-lognormal channel models and the statistical properties for them is given in Sections II, III respectively. After that in Section IV it is shown how the

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deterministic simulation model for Weibull-lognormal is obtained by using the concept of Rice's sum of sinusoids. For this purpose three computation methods of simulation model parameters is presented in Section V. In Section VI, the performance of the three methods is evaluated by comparing statistical properties of reference and simulation models over Weibull-lognormal channel model, where it is observed that new method improved statistical properties performance of simulation models more than other methods, which will be reflected on the performance of the simulator. Finally the conclusion is given in Section VII.

II. Rayleigh Channel Model

Rayleigh and Rice channels are the most important channel models in mobile communications. Usually, Rayleigh and Rice processes are preferred for modelling fast-term fading, whereas slow-term fading is modelled by a lognormal process. Rice process is proposed an appropriate stochastic model for describing the envelope of received signal in rural areas, where the Line of Sight (LOS) is taken into consideration, whereas Rayleigh process is considered suitable for urban regions, where LOS is not exist [1], [9].

A Rayleigh process $\xi(t)$ is obtained by taking the absolute value of the zero-mean complex Gaussian process $\mu(t) = \mu_1(t) + j\mu_2(t)$, i.e.:

$$\xi(t) = |\mu(t)| \tag{1}$$

where $\mu(t)$ represents scattered component in the received signal with uncorrelated real and imaginary parts, and variances $\mathrm{var}\{\mu_i(t)\} = \sigma_0^2,\ i = 1, 2$.

A typical shape for the Doppler Power Spectral Density (PSD) of the complex Gaussian processes is given by the Jakes PSD [7]:

$$S_{\mu\mu}(f) = 2S_{\mu_i\mu_i}(f) = \begin{cases} \dfrac{2\sigma_o^2}{\pi f_{\max}\sqrt{1-(\frac{f}{f_{\max}})^2}}, & |f| \le f_{\max} \\ 0 & |f| > f_{\max} \end{cases} ,i = 1, 2 \tag{2}$$

where $f_{\max}$ denotes the maximum Doppler frequency. Taking the inverse Fourier transform of the Jakes PSD results in the following ACF:

$$r_{\mu\mu}(\tau) = 2r_{\mu_i\mu_i}(\tau) = 2\sigma_o^2 J_0\left(2\pi f_{\max}\tau\right) \tag{3}$$

where $J_0(.)$ is the zeroth-order Bessel function of the first kind. The PDF of Rayleigh process $\xi(t)$, which describes the statistical signal variations, is given by the Rayleigh distribution:

$$P_\xi(x) = \begin{cases} \dfrac{x}{\sigma_o^2} e^{-\frac{x^2}{2\sigma_o^2}}, x \ge 0 \\ 0, x < 0 \end{cases} \tag{4}$$

LCR and ADF of Rayleigh Processes $\xi(t)$, which belong to the statistical properties of the second degree, are very important in assessing the performance of channel simulators. LCR is defined as the rate (crossings per second) at which the envelope $\xi(t)$ crosses a given level r in the positive (or negative) going direction. LCR of Rayleigh process can be represented by [1], [9]:

$$N_\xi(r) = \sqrt{\frac{\alpha}{2\pi}} p_\xi(r) \tag{5}$$

It is obvious from (5) that LCR of Rayleigh process is proportional to its PDF by the constant α which is the reverse curve of ACF $\alpha = -\ddot{r}_{\mu_i\mu_i}(0), i = 1, 2$. In the case of isotropic scattering, where the ACF $r_{\mu_i\mu_i}(\tau)$ is given by (3), the quantity α may be written as:

$$\alpha = 2(\pi\sigma_0 f_{\max})^2 \tag{6}$$

On the other hand, ADF $T_{\xi_-}(r)$ is the mean value of the length of all time intervals over which the envelope $\xi(t)$ remains below a given level r. In general, the ADF is defined by [1], [9]:

$$T_{\xi_-}(r) = \frac{F_{\xi\_}(r)}{N_\xi(r)} \tag{7}$$

where $F_{\xi\_}(r)$ the Cumulative Distribution Function (CDF) of $\xi(t)$ defined by $F_{\xi\_}(r) = p_r[\xi(t) \le r]$.

III. Weibull-Lognormal Channel Model

This model is used to simulate non–uniform scattering with shadow fading, where the first one models the possible scattering non–uniformities of the channel, whereas the second accounts for the slow term variations of the local mean due to shadowing. The Weibull-lognormal process *WL*(*t*) is obtained by multiplying a Weibull process *W*(*t*) by a lognormal one *L*(*t*). The Weibull process results from a Rayleigh one *R*(*t*) as $W(t) = R(t)^{2/\beta}$ and β a parameter expressing the fading severity [10]. As we have seen in the Section II, the Rayleigh process can be obtained by (1), whereas the lognormal process is generated by a real valued Gaussian process $\mu_3(t)$ with zero mean and unit variance as $L(t) = \exp[s\mu_3(t) + m]$, where s and m determine the type of shadowing environment. The stochastic Weibull-lognormal process will be [10]:

$$WL(t) = a \cdot [\mu_1^2(t) + \mu_2^2(t)]^{1/\beta} \cdot \exp[s\mu_3(t)] \tag{8}$$

where $a = \exp(m) \cdot \sigma_0^{2/\beta}$. The amplitude PDF $P_{WL}(\cdot)$ of *WL*(*t*) is given by [10]:

$$P_{WL}(x) = \frac{x^{\beta-1}}{2\sqrt{2\pi} s a^\beta} \int_0^\infty \exp(-\frac{x^\beta z}{2a^\beta}) \exp(-\frac{\ln^2 z}{2\beta^2 s^2}) dz \tag{9}$$

We note from (9) that the PDF of Weibull-lognormal process depends on three parameters (s, a, β), and then this model shows high flexibility and includes Suzuki, Weibull and Rayleigh models as special cases. The LCR of Weibull-lognormal process was approximated in [10] from the assumption of a slowly time varying lognormal process compared to the Weibull one. So LCR is defined by the equation [10]:

$$N_r(x) = \frac{\sqrt{2} f_{\max} x^{\beta/2}}{s\beta a^{\beta/2}} \int_0^\infty \exp(-\frac{x^\beta z^2}{2a^\beta}) \exp(-\frac{2\ln^2 z}{\beta^2 s^2}) dz \quad (10)$$

Now ADF easily can be found by using (10) with CDF given by [10]:

$$F_r(x) = 1 - \frac{1}{\sqrt{2\pi} s\beta} \int_0^\infty \frac{1}{u} \exp(-\frac{x^\beta z}{2a^\beta}) \exp(-\frac{\ln^2 z}{2\beta^2 s^2}) dz \quad (11)$$

IV. Deterministic Simulation Model

An efficient simulator for Weibull-lognormal fading channels is obtained by using the concept of Rice's sum of sinusoids [6]. According to that principle, we replace the stochastic Gaussian processes $\mu_1(t)$, $\mu_2(t)$, and $\mu_3(t)$ of the reference model by the following deterministic processes:

$$\tilde{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}) \qquad i = 1,2,3 \quad (12)$$

where N_i denotes the number of sinusoids. The parameters $c_{i,n}$, $f_{i,n}$, and $\theta_{i,n}$ are called Doppler coefficients, Doppler frequencies and Doppler phases respectively. These parameters have to be computed during the simulation setup phase, e.g., by one of the methods described in the following sections. From the fact that all parameters are known quantities, it follows that $\tilde{\mu}_i(t)$ can be considered as a deterministic process. In order the deterministic process $\tilde{\mu}_i(t), i = 1,2$, to be uncorrelated, we define $N_2 = N_1 + 1$ [1], [9], thus we obtain the three deterministic Gaussian processes $\tilde{\mu}_i(t), i = 1,2,3$. By analogy with (8), the received envelope of Weibull-lognormal fading channels can be modeled according to:

$$\tilde{r}(t) = a \cdot [\tilde{\mu}_1^2(t) + \tilde{\mu}_2^2(t)]^{1/\beta} \cdot \exp[s\tilde{\mu}_3(t)] \quad (13)$$

In general, the ACF of $\tilde{\mu}_1(t)$ and $\tilde{\mu}_2(t)$ are given by [1], [9]:

$$\tilde{r}_{\mu_i\mu_i}(\tau) = \sum_{n=1}^{N_i} \frac{c^2_{i,n}}{2} \cos(2\pi f_{i,n} \tau), \qquad i = 1,2 \quad (14)$$

V. Computation Methods for the Model Parameters

This section presents three different methods for the determination of the Doppler coefficients $c_{i,n}$ and the corresponding discrete Doppler frequencies $f_{i,n}$. The Doppler phases $\theta_{i,n}, i = (1,2,3)$, are realizations of a random variable uniformly distributed within the interval $(0, 2\pi]$ [1], [9]. The procedures will be named by Method of Equal Areas (MEA), Method of Exact Doppler Spread (MEDS), and the new one is named by Combination of MEDS and MEA method. Here, we will not present in detail the simulation modeling employing sum of sinusoids, but for the interested reader we refer to [9] for detailed and well-presented analysis of the main methods used in the sum of sinusoids simulation scheme.

A. *Method of Equal Areas (MEA)*

The Doppler coefficients $c_{i,n}$ have been designed in terms of fulfilling the power constraint $\tilde{\sigma}_0^2 = \sigma_0^2$. The frequencies $f_{i,n}$ can be found by partitioning the Doppler power spectral density of $\tilde{\mu}_i(t)$ into N_i sections of equal power and using the upper frequency limits, related to these areas. The Doppler coefficients $c_{i,n}$ and frequencies $f_{i,n}$ are computed by [1], [9], [12]:

$$c_{i,n} = \sigma_0 \sqrt{2/N_i} \quad (15)$$

$$f_{i,n} = f_{\max} \sin(\frac{\pi n}{2N_i}) \quad (16)$$

respectively for $n = 1,2,..., N_i, i = 1,2$.

B. *Method of Exact Doppler Spread (MEDS)*

The MEDS is documented in [1], [9]. For the computation of the gains $c_{i,n}$, the same is valid as MEA. Applying the MEDS results the frequencies $f_{i,n}$ in the following relation:

$$f_{i,n} = f_{\max} \sin[\frac{\pi}{2N_i}(n - \frac{1}{2})] \quad (17)$$

respectively for $n = 1,2,..., N_i, i = 1,2$.

The discrete frequencies $f_{3,n}$ of $\tilde{\mu}_3(t)$ are calculated with a simple modification of MEDS method by means of the relations [1], [9]:

$$\frac{2n-1}{N_3} - erf(\frac{f_{3,n}}{\sqrt{2}\sigma_c}) = 0, n = 1,2,.., N_3 - 1$$
$$f_{3,N_3} = \sqrt{\sigma_c^2 N_3 - \sum_{n=1}^{N_3-1} f_{3,n}^2} \quad (18)$$

As $\tilde{\mu}_3(t)$ has a Gaussian PSD $S_{\mu_3\mu_3}(f)$ defined as [1], [9]:

$$S_{\mu_3\mu_3}(f) = \frac{1}{\sqrt{2\pi}\sigma_c} \exp(-\frac{f^2}{2\sigma_c^2}) \quad (19)$$

where σ_c a parameter related to the 3dB-cut-off frequency f_c according to $f_c = \sigma_c\sqrt{2\ln 2}$, whereas f_c is much smaller than the maximum Doppler frequency $f_{\max}$, i.e. $f_c << f_{\max}$, so the frequency ratio $K_c = f_{\max}/f_c$ is much greater than one, i.e. $K_c >> 1$. In real worlds channels $K_c > 10$ [1].

C. Combination of MEDS and MEA

The new method relies on the application of both methods MEDS, MEA, but we must apply the method MEDS on the first half number of sinusoids $N_i/2$. Therefore, c_{i,n_1} and f_{i,n_1} are given as follows:

$$c_{i,n_1} = \sigma_0 \sqrt{\frac{2}{N_i}} \tag{20}$$

$$f_{i,n_1} = f_{\max} \sin[\frac{\pi}{2N_i}(n_1 - \frac{1}{2})] \tag{21}$$

where $n_1 = 1, 2, ..., N_i/2$, $i = 1, 2$, then MEA method is applied on the second half of sinusoids number:

$$c_{i,n_2} = \sigma_0 \sqrt{\frac{2}{N_i}} \tag{22}$$

$$f_{i,n_2} = f_{\max} \sin[\frac{\pi}{2N_i}(n_2 - \frac{1}{2})] \tag{23}$$

where $n_2 = (N_i/2)+1, (N_i/2)+2, ..., N_i$, $i = 1, 2$. Finally the formulas for $c_{i,n}$ and $f_{i,n}$ according to combination of MEDS and MEA are given by $c_{i,n} = [c_{i,n_1}, c_{i,n_2}]$ and $f_{i,n} = [f_{i,n_1}, f_{i,n_2}]$ respectively.

VI. Comparison of Statistical Properties between Reference and Simulation Models

The statistical properties of the reference model for Weibull-lognormal fading channel are compared with the corresponding simulation results. Assuming that simulation model parameters have been found in one of the previously described methods, In this case, the parameters in (12) are known quantities and the ACF $\tilde{r}_{\mu_i\mu_i}(\tau)$ of the simulation model can be calculated for $i = 1, 2$ by means of (14), whereas the ACF $r_{\mu_i\mu_i}(\tau)$ of the reference model is obtained from (3). Both of ACFs $r_{\mu_i\mu_i}(\tau)$, $\tilde{r}_{\mu_i\mu_i}(\tau)$ are compared with $N_i = 25, i = 1, 2$ through Fig. 1, Fig. 2, and Fig. 3, where the computation of the simulation model parameters was based on MEDS, MEA, and combination of MEDS and MEA, respectively. It is observed that fitting between ACFs is excellent at least up to the $N_i th$ zero-crossing of $r_{\mu_i\mu_i}(\tau)$ for all methods, but the Fig. 3 shows the error has become less between the reference and simulation models, especially in the last three sinusoidal harmonics of the ACF in comparison with Fig. 1 and Fig. 2. This means that ACFs of both reference and simulation models are much closer according to the new method than MEA and MEDS, and of course this will affects the statistical properties later when LCR and ADF are studied for Weibull-lognormal model. From now the number of sinusoids was assumed $N_1 = N_3 = 15$ and $N_2 = 16$ for Weibull-lognormal fading channel. The Fig. 4 shows the PDF of the received envelope of Weibull-lognormal fading channel as a function of K_c, with a parameter set defined as β=2.8, s = 0.5, $m = 1$, $\sigma_0 = 0.8$, $f_{max} = 91Hz$. From Fig. 4 an excellent conformity between the reference and simulation PDF's is revealed, no matter what the value of K_c is. Because PDF of Weibull-lognormal process is independent of time selectivity of the lognormal process. In Fig. 5, Fig. 6, and Fig. 7, the normalized LCR of deterministic Weibull-lognormal processes for all introduced methods with $K_c = 1$, $K_c = 2$, and $K_c = 3$ is shown, respectively. It can be observed clearly that LCR of the simulation model is very close to that of the reference model according to the combination of MEDS and MEA in comparison with MEA and MEDs. The corresponding graphs for the normalized ADF are presented in Fig. 8, Fig. 9, and Fig. 10. In addition increasing K_c improves the performance of LCR and ADF of the simulation model. Finally, the results documented in the Fig. 5-Fig. 10 indicate a superiority of the combination of MEDS and MEA over the MEDS and MEA with respect to LCR and ADF.

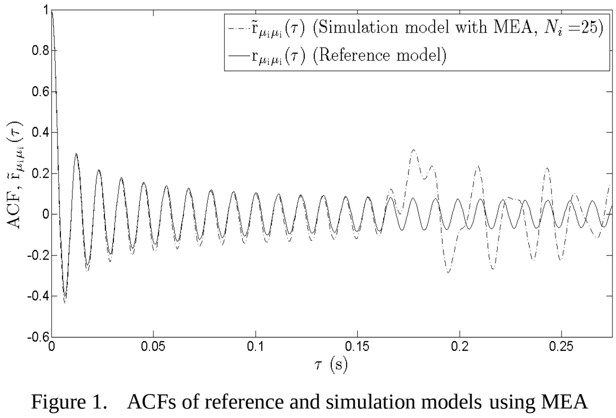

Figure 1. ACFs of reference and simulation models using MEA

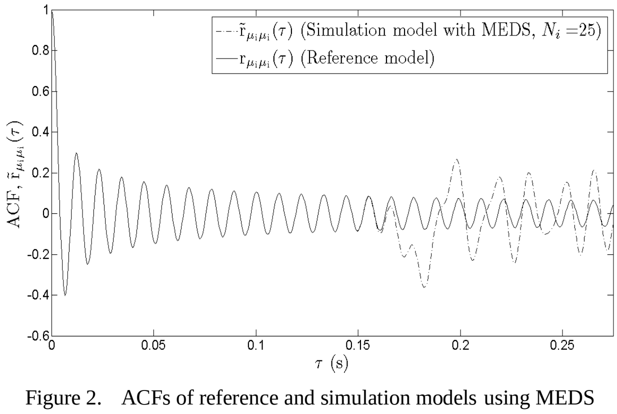

Figure 2. ACFs of reference and simulation models using MEDS

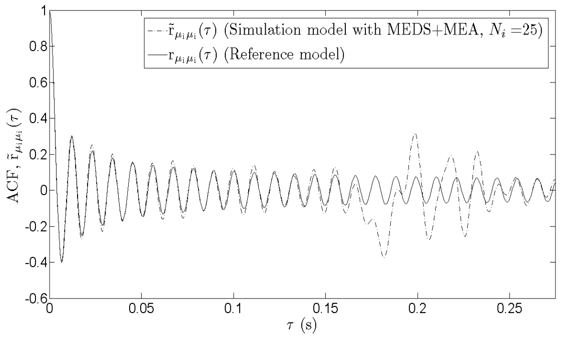

Figure 3. ACFs of reference and simulation models using combination of MEDS and MEA

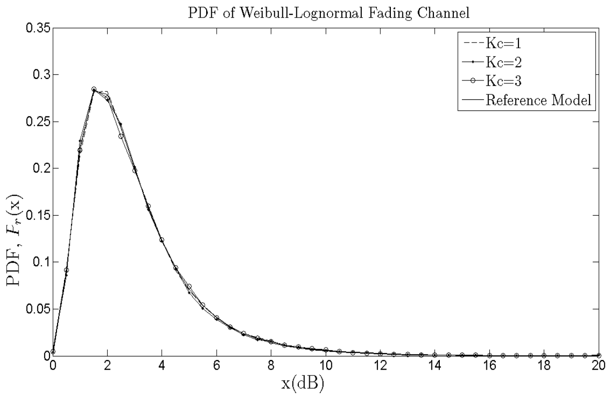

Figure 4. The PDF of deterministic Weibull-lognormal processes with variation of K_c

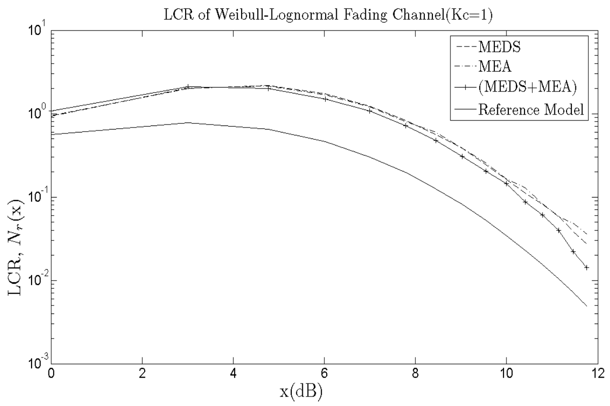

Figure 5. The LCR of deterministic Weibull-lognormal processes using MEDS, MEA, combination of MEDS and MEA with $K_c = 1$

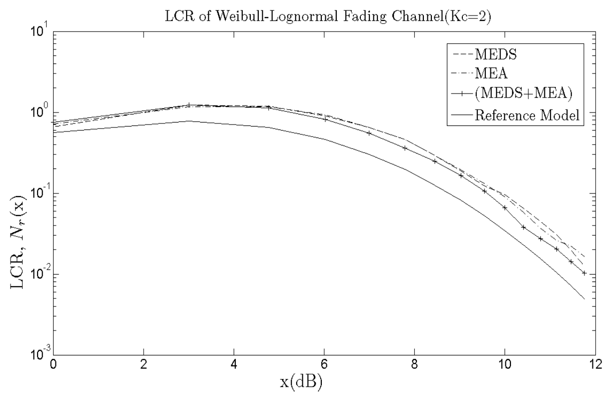

Figure 6. The LCR of deterministic Weibull-lognormal processes using MEDS, MEA, combination of MEDS and MEA with $K_c = 2$

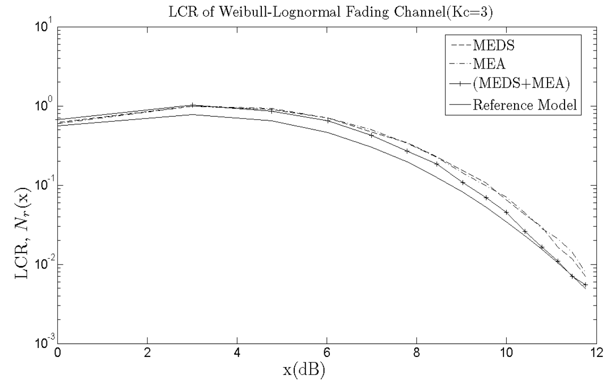

Figure 7. The LCR of deterministic Weibull-lognormal processes using MEDS, MEA, and combination of MEDS and MEA with $K_c = 3$

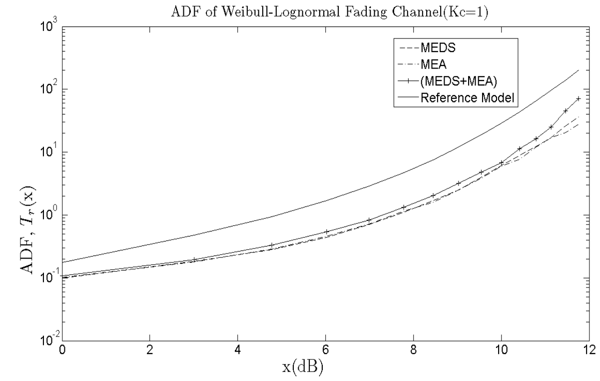

Figure 8. The ADF of deterministic Weibull-lognormal processes using MEDS, MEA, and combination of MEDS and MEA with $K_c = 1$

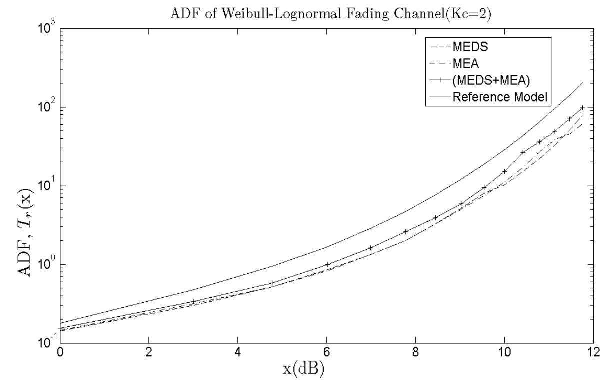

Figure 9. The ADF of deterministic Weibull-lognormal processes using MEDS, MEA, and combination of MEDS and MEA with $K_c = 2$

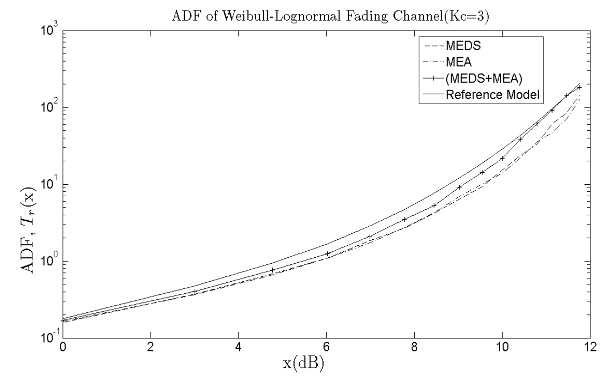

Figure 10. The ADF of deterministic Weibull-lognormal processes using MEDS, MEA, and combination of MEDS and MEA with $K_c = 3$

VII. Conclusion

The concept of Rice's sum of sinusoids is used to design an efficient deterministic simulation models for Weibull-Lognormal fading channels. A study of the statistics of such types of simulation models was the topic of the present paper, especially for the PDF, LCR, and ADF. New computation method of deterministic simulation model parameters is presented, this method is called a combination of MEA and MEDS. The performance of different parameter computation methods is discussed and evaluated by comparing ACFs of both reference and simulation models. It is observed that ACFs of reference and simulation models are much closer according to the new method than MEA and MEDS. In addition, the new method gave us an excellent results corresponding with PDF, LCR, and ADF of deterministic simulation model. Therefore, the deterministic simulation model, based on the combination of MEDS and MEA, will be very close in its statistical properties to those of the reference model. Finally, the improved performance of the statistical properties of deterministic simulation fading channel models lead to get a high accuracy and efficiency fading channel simulator.

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