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# Improve the Performance of Statistical Properties for Deterministic Channel Simulators

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**Abstract** — This paper evaluates statistical properties of Rice channel model, such as Autocorrelation Function (ACF), Level Crossing Rate (LCR), Average Duration of Fades (ADF), and Cumulative Distribution Function (CDF). New computation procedure of deterministic simulation model parameters is presented. This procedure is a combination of two methods, Method of Equal Areas (MEA) and Method of Exact Doppler Spread (MEDS). It is called a combination of MEA and MEDS. Comparisons of statistical properties for both reference and simulation models are introduced. Finally, the results indicate a superiority of the new method over the MEDS and MEA with respect to LCR and ADF.

**Keywords** — Method of Equal Areas, Level Crossing Rate, Average Duration of Fades, Method of Exact Doppler Spread.

## I. INTRODUCTION

The modeling of fading channels is of great importance in the design, test and improve the performance of cellular radio communication systems. But the channel simulator must be efficient, flexible and accurate. That depends on the design method of simulator. The algorithm of channel simulator should be simple to implement on the computer. Depending upon the radio propagation environment various multipath fading models are available in literature [1], whereas [2] presents an explanation for many classical fading channel models presented since 2005 until present. Mobile fading channels are classified into two categories, namely: frequency–nonselective and frequency–selective fading channels. The first type is modeled by using an appropriate stochastic models, such as Rayleigh, Rice and Suzuki processes. These processes play an important role in modelling mobile fading channels with a different degree of complexity. Frequency–selective channels can be modelled by using (n-path) tap delay line model [3]. This model requires  $2n$  coloured Gaussian processes. Therefore, computer simulation models can be implemented by means of Rice method [4], which depend on approximation of the coloured Gaussian processes by finite sum of weighted sinusoids with phases uniformly distributed. Finding proper design method for computing parameters of simulation models,

provides deterministic processes at the output of channel simulator with a statistical properties closed to those of the corresponding stochastic processes. Especially statistics of fading time intervals known as level-crossing problem [5]. Level-crossing rate (LCR) and average duration of fades (ADF) are a statistical properties of second order. Analytical expressions for these quantities have been derived for Rayleigh [6], Rice [7]. The Statistics of deterministic processes are similar (identical) to those for the reference model if the number of sinusoids is sufficient (infinite). There are many methods to calculate the parameters of simulation model (doppler coefficients and discrete doppler frequencies), for example, the Method of Equal Areas (MEA) [8], which provides a satisfied approximation of the desired statistics for Jakes Doppler power spectral density even for a small number of sinusoids, but it fails or requires large number of sinusoids for other types of Doppler power spectral densities such as those with Gaussian shapes [9]. The MEA does not result in a periodic autocorrelation function (ACF) due to unequal distances between discrete Doppler frequencies [10]. Method of exact Doppler spread (MEDS) is another method used to compute parameters in such way the Doppler spread is the same for both reference and simulation models [11]. A new design method for calculating parameters of simulation model is presented. This method is named a combination of MEA and MEDS. The performance of the three methods is evaluated by comparing ACF of reference and simulation models because ACF is related to LCR and ADF quantities. The statistics of reference and simulation models are evaluated over Rice channel model. It is noticed by simulation results that new method improved statistical properties performance of simulation model more than other methods.

## II. DESCRIPTION OF RICE REFERENCE MODEL

Rayleigh and Rice channels are the most important channel models in mobile communications. They are comparatively easy to describe, and they can be implemented in software and hardware very efficiently and with a high degree of precision [12]. Usually, Rayleigh and Rice processes are preferred for modelling fast-term fading, whereas slow-term fading is modelled by a lognormal process [12,13]. The Rice

model is often applicable in an indoor environment, whereas the Rayleigh model characterizes outdoor settings. The Rice model also becomes more applicable in smaller cells or in more open outdoor environments [14]. A Rice process  $\xi(t)$  is obtained by taking the absolute value of the nonzero-mean complex Gaussian process  $\mu_p(t) = \mu_1(t) + j\mu_2(t) + m(t)$ , i.e. [12,13]:

$$\xi(t) = |\mu_p(t)| \quad (1)$$

The zero-mean complex Gaussian random process  $\mu(t) = \mu_1(t) + j\mu_2(t)$  represents scattered component in the received signal with uncorrelated real and imaginary parts, and variances  $Var\{\mu(t)\} = 2Var\{\mu_i(t)\} = 2\sigma_0^2, i = 1, 2$ .

In the following, the line-of-sight (LOS) component of the received signal will be described by a complex sinusoid of the form [12,13]:

$$m(t) = m_1(t) + jm_2(t) = \rho e^{j(2\pi f_\rho t + \theta_\rho)}$$

where  $\rho$ ,  $f_\rho$ , and  $\theta_\rho$  denote the amplitude, the Doppler frequency, and the phase of LOS component, respectively. A typical shape for the Doppler power spectral density (PSD) of the complex Gaussian processes is given by the Jakes PSD [5,12,13]:

$$S_{\mu_i}(f) = \begin{cases} \frac{2\sigma_0^2}{\pi f_{\max} \sqrt{1 - (\frac{f}{f_{\max}})^2}}, & |f| \leq f_{\max} \\ 0 & |f| > f_{\max} \end{cases}$$

where  $f_{\max}$  denotes the maximum Doppler frequency. Taking the inverse Fourier transform of the Jakes PSD results in the following ACF [12,13]:

$$r_{\mu_i}(\tau) = 2\sigma_0^2 J_0(2\pi f_{\max} \tau) \quad (4)$$

where  $J_0(\cdot)$  is the zeroth-order Bessel function of the first kind. The probability density function (PDF) of Rice process  $\xi(t)$  is given by [12,13]:

$$P_\xi(x) = \frac{x}{\sigma_0^2} e^{-\frac{x^2 + \rho^2}{2\sigma_0^2}} I_0\left(\frac{x\rho}{\sigma_0^2}\right), x \geq 0$$

where  $I_0(\cdot)$  designates the zeroth-order modified Bessel function of the first kind. If the LOS component does not exist, the Rice process  $\xi(t)$  results in the Rayleigh process  $\zeta(t)$ , whose statistical signal variations are described by the Rayleigh distribution [12,13]:

$$P_\zeta(x) = \begin{cases} \frac{x}{\sigma_0^2} e^{-\frac{x^2}{2\sigma_0^2}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The cumulative distribution function (CDF) of  $\xi(t)$  defined by  $F_{\xi^-}(r) = p_r[\xi(t) \leq r]$  can be expressed by [15]:

$$F_{\xi^-}(r) = 1 - Q\left(\frac{\rho}{\sigma_0}, \frac{r}{\sigma_0}\right) \quad (7)$$

where  $Q(\cdot, \cdot)$  is the Marcum function.

The LCR and ADF of Rice Processes  $\xi(t)$  belong to the statistical properties of the second degree, are very important in assessing the performance of channel simulators, They will be denoted here by  $N_\xi(r)$  and  $T_{\xi^-}(r)$ , respectively. LCR is defined as the rate (in crossings per second) at which the envelope  $\xi(t)$  crosses the pre-given level  $r$  in the positive (or negative) going direction. LCR of Rice process can be represented by [12,13]:

$$N_\xi(r) = \sqrt{\frac{\alpha}{2\pi}} p_\xi(r) \quad (8)$$

It is obvious from (8) that LCR of Rice process is proportional to its PDF  $p_\xi(r)$  by the constant  $\alpha$  which is the reverse curve of ACF  $\alpha = -\ddot{r}_{\mu_i \mu_i}(0), i = 1, 2$ . For the case of isotropic scattering, where the ACF  $r_{\mu_i \mu_i}(\tau)$  is given by (4), the quantity  $\alpha$  may be written as [12,13]:

$$\alpha = 2(\pi\sigma_0 f_{\max})^2 \quad (9)$$

The ADF  $T_{\xi^-}(r)$  is the mean value for the length of all time intervals over which the envelope  $\xi(t)$  remains below a given level  $r$ . In general, the ADF is defined by [12,13]:

$$T_{\xi^-}(r) = \frac{F_{\xi^-}(r)}{N_\xi(r)} \quad (10)$$

### III. DETERMINISTIC RICE SIMULATION MODEL

An efficient simulator for the Rice fading channels is obtained by using the concept of Rice's sum of sinusoids [4]. According to this principle, we replace the zero-mean Gaussian processes  $\mu_1(t)$  and  $\mu_2(t)$  of reference model by [12,13]:

$$\tilde{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}) \quad i = 1, 2 \quad (11)$$

where  $N_i$  denotes the number of sinusoids. Thus our task is to simulate the above two processes in such a way that the first and second order statistics of both models (reference and simulation models) are as close as possible (ideally identical). The parameters  $c_{i,n}$ ,  $f_{i,n}$ , and  $\theta_{i,n}$  are called Doppler coefficients, Doppler frequencies and Doppler phases of simulation model, respectively. By analogy with (1), the received

envelope of Rice fading channel can be modeled according to:

$$\tilde{\xi}(t) = |\tilde{\mu}_p(t)|$$

In general, the ACF of  $\tilde{\mu}_1(t)$  and  $\tilde{\mu}_2(t)$  are given by [12,13]:

$$\tilde{r}_{\mu_i \mu_i}(t) = \sum_{n=1}^{N_i} \frac{c_{i,n}^2}{2} \cos(2\pi f_{i,n} t), \quad i = 1, 2$$

In the next section, we present a new computation method for simulation model parameters. For reasons of comparison, we also investigate the statistical properties of simulation model for two other parameter computation methods.

#### IV. COMPUTATION METHODS FOR SIMULATION MODEL PARAMETERS

We present three different methods for the determination of the Doppler coefficients  $c_{i,n}$  and the corresponding discrete Doppler frequencies  $f_{i,n}$ . The Doppler phases  $\theta_{i,n}, i = (1, 2, 3)$ , are realizations of a random variable uniformly distributed within the interval  $(0, 2\pi]$  [12,13]. The procedures are MEA, MEDS, and the new one is named by Combination of MEDS and MEA. Here we will not present in detail the simulation modeling employing sum of sinusoids. But for the interested reader we refer to [12,13] for detailed and well-presented analysis of the main methods used in the sum of sinusoids simulation scheme.

##### A. Method of Equal Areas (MEA)

The gains  $c_{i,n}$  have been designed in terms of fulfilling the power constraint  $\tilde{\sigma}_0^2 = \sigma_0^2$ . The frequencies  $f_{i,n}$  can be found by partitioning the Doppler power spectral density of  $\tilde{\mu}_i(t)$  into  $N_i$  sections of equal power and using the upper frequency limits, related to these areas. The gains  $c_{i,n}$  and frequencies  $f_{i,n}$  are computed by [10,12,13]:

$$c_{i,n} = \sigma_0 \sqrt{\frac{2}{N_i}}$$

$$f_{i,n} = f_{\max} \sin\left(\frac{\pi n}{2N_i}\right)$$

respectively for  $n = 1, 2, \dots, N_i, i = 1, 2$ .

##### B. Method of Exact Doppler Spread (MEDS)

The MEDS is documented in [12,13]. For the computation of the gains  $c_{i,n}$ , the same is valid as it was described for the previous method. The frequencies  $f_{i,n}$  are determined in such a way that the Doppler spread of the simulation model is exactly equal to the Doppler spread of the reference model for

any number of sinusoids  $N_i$ . The formulas for  $c_{i,n}$  and  $f_{i,n}$  by applying the MEDS are given by [12,13]:

$$c_{i,n} = \sigma_0 \sqrt{\frac{2}{N_i}} \tag{15a}$$

$$f_{i,n} = f_{\max} \sin\left[\frac{\pi}{2N_i}\left(n - \frac{1}{2}\right)\right] \tag{15b}$$

respectively, for  $n = 1, 2, \dots, N_i, i = 1, 2$ .

##### C. Combination of MEDS and MEA

The new method relies on the application of both methods MEDS, MEA, but we must apply the method MEDS on the first half number of sinusoids  $N_i/2$ .

Therefore  $c_{i,n}$  and  $f_{i,n}$  are given as follows:

$$c_{i,n1} = \sigma_0 \sqrt{\frac{2}{N_i}} \tag{16a}$$

$$f_{i,n1} = f_{\max} \sin\left[\frac{\pi}{2N_i}\left(n - \frac{1}{2}\right)\right] \tag{16b}$$

where  $n = 1, 2, \dots, N_i/2, i = 1, 2$ , then MEA method is applied on the second half of sinusoids number:

$$c_{i,n2} = \sigma_0 \sqrt{\frac{2}{N_i}} \tag{17a}$$

$$f_{i,n2} = f_{\max} \sin\left[\frac{\pi}{2N_i}\left(n - \frac{1}{2}\right)\right] \tag{17b}$$

where  $n = (N_i/2) + 1, (N_i/2) + 2, \dots, N_i, i = 1, 2$ .

Finally we get the formulas for  $c_{i,n}$  and  $f_{i,n}$  according to combination of MEDS and MEA by  $c_{i,n} = [c_{i,n1}; c_{i,n2}]$  and  $f_{i,n} = [f_{i,n1}; f_{i,n2}]$  respectively.

#### V. COMPARISON OF STATISTICAL PROPERTIES FOR BOTH REFERENCE AND SIMULATION MODELS

This The statistical properties of reference model for Rice fading channel are compared with the corresponding simulation results by using equations (14a, 8, 10). Assuming that simulation model parameters have been found in one of the previously described methods, In this case, the parameters are known quantities and the ACF  $\tilde{r}_{\mu_i \mu_i}(\tau)$  of simulation model can be calculated for  $i = 1, 2$  by means of (13), whereas ACF  $r_{\mu_i \mu_i}(\tau)$  of reference model is obtained from (4). Both ACFs  $r_{\mu_i \mu_i}(\tau), \tilde{r}_{\mu_i \mu_i}(\tau)$  are compared with  $N_i = 12, i = 1, 2$  through Fig. 1, Fig. 2, and Fig. 3, where the computation of simulation model parameters was based on MEDS, MEA, and combination of MEDS and MEA, respectively.

Fig. 4, Fig. 5, and Fig. 6 show comparisons of ACFs with  $N_i = 25, i=1,2$ , we observe that  $r_{\mu_i, \mu_i}(\tau) \approx \tilde{r}_{\mu_i, \mu_i}(\tau)$  if  $\tau \in [0, 0.15]$  [sec] for all methods, but Fig. 3 and Fig. 6 show that the error has become less between the reference and simulation models, especially in the last three sinusoidal harmonics of ACF in comparison with MEA and MEDS. This means that ACFs of both reference and simulation models are more closer according to the new method than MEA and MEDS, and of course this result will affect the statistical properties later when LCR and ADF are evaluated. Throughout the following section the number of sinusoids was assumed  $N_1 = 8$  and  $N_2 = 9$  for the deterministic Gaussian processes of the Rice (Rayleigh) fading channel. The CDF of deterministic Rice (Rayleigh) processes is shown in Fig. 7. It is noticed that the simulation results in Fig. 4 are very close to the CDF of reference model for all used methods over Rice fading channel model. This is not surprising, because the MEA and MEDS use the same procedure concerning the gains  $c_{i,n}$ , which is, as mentioned above, i.e.,  $\tilde{\sigma}_0^2 = \sigma_0^2$ . Another observation from Fig. 7, it has been shown that the performance of a combination of MEDS and MEA according to the CDF for both reference and simulation models, is better than MEDS, MEA. In Fig. 8, the normalized LCR of deterministic Rice (Rayleigh) processes for all introduced methods is shown, it can be observed that the LCR of simulation model is very close to that of reference model according to the combination of MEDS and MEA. Finally, we present the corresponding graphs for the ADF in Fig. 9. The results documented in the figures (7-9) indicate a superiority of the combination of MEDS and MEA over the MEDS and MEA with respect to LCR and ADF.

**VI. CONCLUSION**

The concept of Rice’s sum of sinusoids enables an efficient design for Rice (Rayleigh) simulation models. A study of statistical properties of such types of simulation models was the topic of the present paper. Especially for the CDF, LCR, and ADF. New computation method of deterministic simulation model parameters is presented, this method is a combination of MEA and MEDS. We discussed and evaluated the performance of different parameter computation methods by comparing ACFs of both reference and simulation models. It is observed that ACFs of reference and simulation models are more closer according to the new method than MEA and MEDS. In addition, the new method gave us an excellent results corresponding with CDF, LCR, and ADF of deterministic simulation model, therefore, we can say that the deterministic simulation model, based on the combination of MEDS and MEA, will be very close in its statistical properties to the reference

model. Finally, the improved performance of statistical properties of deterministic simulation fading channel models lead to get a high accuracy and efficiency fading channel simulator.

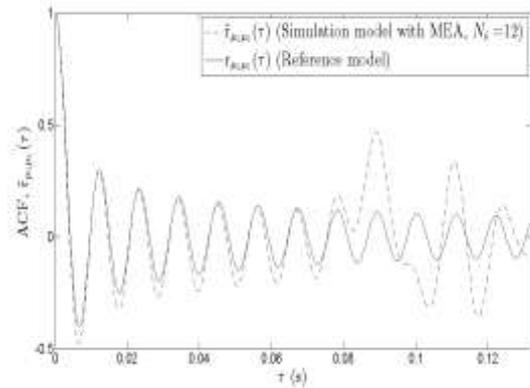


Fig. 1 ACFs of reference and simulation models using MEA

(Ni=12, fmax = 91 Hz,  $\sigma_0^2 = 1$ )

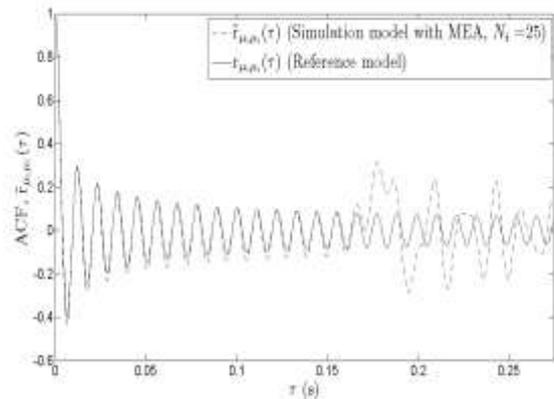


Fig. 2 ACFs of reference and simulation models using MEA

(Ni=25, fmax = 91 Hz,  $\sigma_0^2 = 1$ )

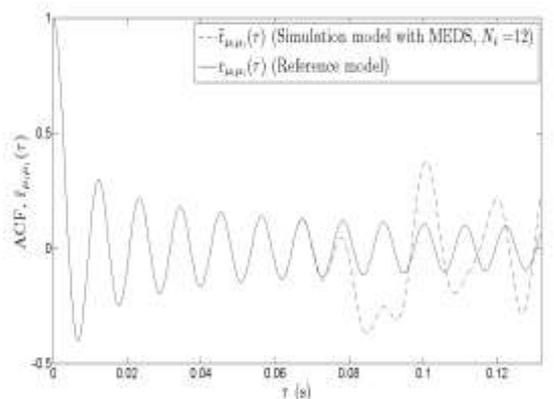


Fig. 3 ACFs of reference and simulation models using MEDS

(Ni=12, fmax = 91 Hz,  $\sigma_0^2 = 1$ )

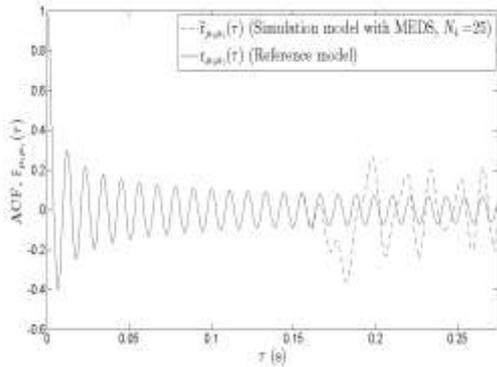


Fig. 4 ACFs of reference and simulation models using MEDS  
( $N_i=25$ ,  $f_{max} = 91$  Hz,  $\sigma_0^2 = 1$ )

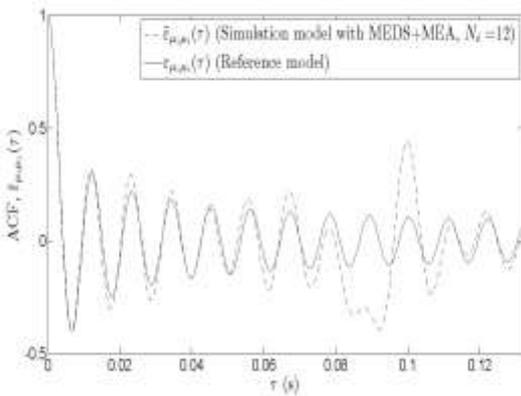


Fig. 5 ACFs of reference and simulation models using MEDS+MEA  
( $N_i=12$ ,  $f_{max} = 91$  Hz,  $\sigma_0^2 = 1$ )

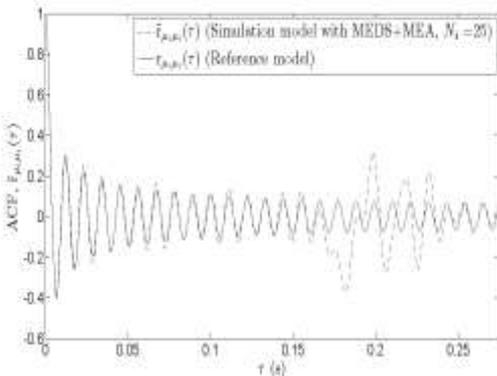


Fig. 6 ACFs of reference and simulation models using MEDS+MEA  
( $N_i=25$ ,  $f_{max} = 91$  Hz,  $\sigma_0^2 = 1$ )

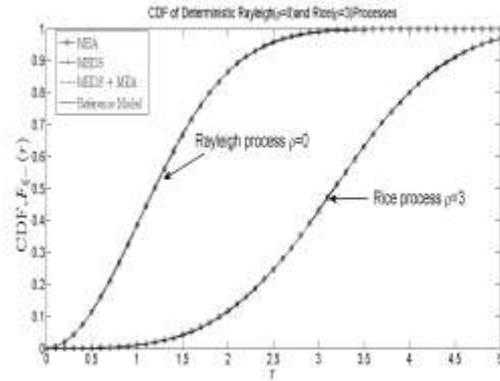


Fig. 7 Comparison of CDF for deterministic Rayleigh ( $\rho = 0$ ) and Rice ( $\rho = 3$ ) processes with ( $N_1 = 8, N_2 = 9, \sigma_0^2 = 1$ )

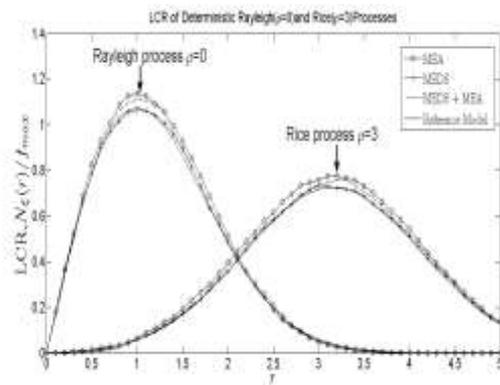


Fig. 8 Comparison of LCR for deterministic Rayleigh ( $\rho = 0$ ) and Rice ( $\rho = 3$ ) processes with ( $N_1 = 8, N_2 = 9, \sigma_0^2 = 1$ )

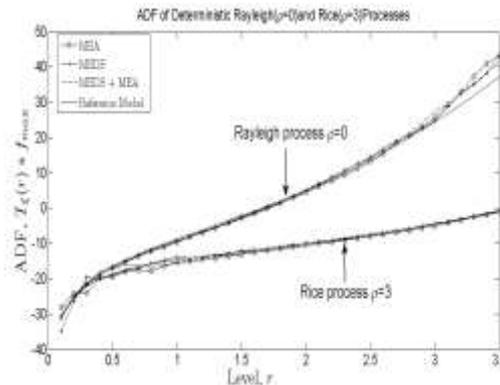


Fig. 9 Comparison of ADF for deterministic Rayleigh ( $\rho = 0$ ) and Rice ( $\rho = 3$ ) processes with ( $N_1 = 8, N_2 = 9, \sigma_0^2 = 1$ )

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